1.9 – Compositions of Matrix Transformations

Definitions: The **composition of T_B with T_A** is achieved by first applying the matrix transformation T_A to a vector and then applying the matrix transformation T_B to the image vector. We denote the composition of T_B with T_A by $T_B \circ T_A$ which is read "T_B circle T_A." This is also expressed as $(T_B \circ T_A)(\mathbf{x}) = T_B(T_A(\mathbf{x}))$.

Theorem 1.9.1 If $T_A : R^n \to R^k$ and $T_B : R^k \to R^m$ are matrix transformations, then $T_B \circ T_A$ is also a matrix transformation, and $T_B \circ T_A = T_{BA}$.

#8 Find the standard matrix for the stated composition in R^2.
a. A rotation about the origin of 60°, followed by an orthogonal projection onto the x-axis, followed by a reflection about the line $y = x$.
b. An orthogonal projection onto the x-axis, followed by a rotation about the origin of 45°, followed by a reflection about the y-axis.
c. A rotation about the origin of 15°, followed by a rotation about the origin of 105°, followed by a rotation about the origin of 60°.

#12 Let $T_1(x_1, x_2, x_3) = (4x_1, -2x_1 + x_2, -x_1 - 3x_2)$ and
$T_2(x_1, x_2, x_3) = (x_1 + 2x_2, -x_3, 4x_1 - x_3)$.

a. Find the standard matrices for T_1 and T_2.

b. Find the standard matrices for $T_2 \circ T_1$ and $T_1 \circ T_2$.

c. Use the matrices obtained in part (b) to find formulas for $T_1(T_2(x_1, x_2, x_3))$ and $T_2(T_1(x_1, x_2, x_3))$.

Definition: If $T_A : R^n \to R^n$ is a matrix operator whose standard matrix A is invertible, then T_A is **invertible**, and the **inverse** of T_A is $T_A^{-1} = T_{A^{-1}}$.

#20 Determine whether the matrix operator $T : R^3 \to R^3$ defined by the equations is invertible; if so, find the standard matrix for the inverse operator, and find $T^{-1}(w_1, w_2, w_3)$.

a. $$\begin{aligned} w_1 &= x_1 - 2x_2 + 2x_3 \\ w_2 &= 2x_1 + x_2 + x_3 \\ w_3 &= x_1 + x_2 \end{aligned}$$

$$w_1 = x_1 - 3x_2 + 4x_3$$

b. $$w_2 = -x_1 + x_2 + x_3$$

$$w_3 = \quad -2x_2 + 5x_3$$